Establishing Traceability for Quantities Derived from Multiple Traceable Quantities

White Paper

Abstract*—Some quantities can each be simply traceable to one single quantity with the same unit but lower uncertainty. For instance, a weight of 50 g ± 1 g might be calibrated with another weight of 50 g ± 0.1 g. Meanwhile, some quantities each can be derived from multiple quantities. A simple example would be measuring DC current with a volt meter and a standard resistor. In this case the DC current is derived from DC voltage and resistance. A more complicated example of a derived quantity is phase noise. The question is: how do we establish traceability for such derived quantities?*

Starting with introducing the concepts of SI, base and derived quantities, and traceability, this paper presents the general approach of establishing traceability for a quantity which is derived from multiple traceable quantities. A measurement procedure will be required, including measurement method, measurement setup and connections, measurement equation, uncertainty analysis, etc. The traceability of phase noise is then considered as an example based upon two common measurement techniques - with a spectrum analyzer and with a phase detector.

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Introduction

There are many quantities in our world, such as length, voltage and phase noise, and even more units for their quantitative descriptions. Among all of the units, the *International System of Units* (SI) [1] defines those which are "*universally agreed for the multitude of measurements that support today's complex society*". Furthermore, seven of them are defined as "*base units*", and the seven quantities they belong to are called "*base quantities*", as listed in Table 1 below. By contrast, other SI units are defined as "*derived units*" which can be expressed as products of powers of base units and their corresponding quantities are similarly called "*derived quantities*". In addition, there are also non-SI units. Some of them are accepted for use with SI (e.g., hour and dB), and others are not recommended for use in modern scientific and technical work (e.g., foot which is 0.3048 m and Chinese "*jin*" which is 0.5 kg).

The choice of each of the base units has been made with special care so that they are independent, "*are readily available to all, are constant throughout time and space, and are easy to realize with high accuracy*" [1], because they provide the foundation for the whole system.

Table 1. SI base quantities and base units.

Base quantity		Base unit	
Name	Symbol	Name	Symbol
Length	$l, x, r,$ etc.	Meter	m
Mass	m	Kilogram	kg
Time, Duration	t	Second	s
Electric current	I, i	Ampere	A
Thermodynamic temperature	T	Kelvin	K
Amount of substance	n	Mole	mol
Luminous intensity	I_v	Candela	cd

However, while the base quantities and units are the basis of the SI system, not all quantities are necessarily traced to them; being traceable to derived quantities can be enough. One practical reason is calibration laboratories don't have to manifest traceability all the way to the highest level, but just to the quantities that are readily traceable. Besides, even at the top of traceability chains, not every base quantity has an intrinsic standard providing the highest accuracy. A typical example is as described in the Appendix 2 of the SI brochure [2] and Section 2.1.5 of NCSLI *Catalog of Intrinsic and Derived Standards* [3]. The high accuracy of electric current is realized through the combination of the realizations of voltage and resistance. In this case the base quantity is derived from (and traced to) the derived quantities. In general, on a traceability chain it doesn't matter if a quantity is a base one or a derived one.

It is accuracy that does matter. The general image of a traceability chain is that a quantity is more accurate at higher levels and less accurate at lower levels. For example, an iron weight of 50 g ± 1 g can be calibrated with and thus traced to another alloy steel weight of 50 g ± 0.1 g. The accuracy relationship is simple and obvious. However, some quantities are not each traced to the same as itself with higher accuracy but derived from several others. Their traceability can still be established, although it may be much more complicated.

In next section of this paper, the general approach of establishing traceability for such a quantity which is derived from multiple traceable quantities is introduced. The complex derived quantity of phase noise is then reviewed as an example.

2. General Approach of Establishing Traceability for Derived Quantities

Traceability, or so-called "*metrological traceability*" in VIM (term 2.41), is defined as "*property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the measurement uncertainty*" [4].

The VIM definition discloses that the establishment of traceability is through quantitative description of the measurement technique, i.e., the measurement uncertainty (MU). Thus the approach of establishing traceability is actually sequential translations from the physical principle of the measurement (calibration) into a calculated number. For this purpose, below are the necessary contents following the translation sequence.

The first thing that should be in place is the measurand - the derived quantity of interest, including the ranges to be determined. If the quantity is of a particular instrument, it should be specified. Sometimes the concept of the quantity and other aspects which may confuse the quantity also need be clarified, depending upon the complexity of the quantity.

Next is the measurement method, including the measurement principle, measuring and test equipment (M&TE) and ancillary apparatus used and step-by-step procedure of connections, settings, data processing, etc.

Thirdly are the measurement equation(s) and uncertainty equation(s) which relate the derived quantity of interest to other traceable ones. The uncertainty equations are generally based on measurement equations but much more complicated due to the inclusion of all the MU contributors of all the traceable quantities. Usually the final measurement result is calculated from several intermediate data and the final MU can only be calculated from a set of equations. The physical principle of the measurement method is translated here into an algebraic format.

Finally, the values and uncertainties of the traceable parameters and the specifications of the M&TEs are substituted into the uncertainty equation(s) and the MU of the derived parameter can be obtained. However, to ensure the validity of the substituted numbers, other supporting evidences are also needed, such as the environmental conditions for storing the M&TEs and performing the measurement, the stabilization of the measurement setup, the validation method of the measurement results, as well as their management.

Among all of the content above, the measurement principle is the fundamental of the whole approach and most attention should be paid to it because misunderstanding the physical principle could result in significant MU contributors being either omitted or by contrast over estimated. Following this idea the next section, where the traceability of phase noise is held as an example, focuses on the physical principle of the measurement and doesn't detail the mathematical calculations.

3. Traceability of Phase Noise - an Example

3.1. Phase Noise Overview

Phase noise is one of the most significant characteristics of a signal generating device and can well be a limiting factor in its use. One of the key specifications of an oscillating source is its capability to produce the same frequency throughout a specific period of time, i.e., the frequency stability. Phase noise describes in frequency domain the short-term frequency stability of less than a few seconds.

Historically, the most generally used unit of measure for phase noise has been the single sideband (SSB) power within a 1 Hz bandwidth at a frequency f offset from the carrier, referenced to the carrier frequency power. The unit of measure is represented as script $\mathscr{L}(f)$ in units of dBc/Hz, as shown by Equation 1 and Figure 1 [5].

$$\mathscr{L}(f) = 10\log\left(\frac{\text{Noise Power in 1 Hz BW}}{\text{Total Power in Full BW}}\right) \qquad \textit{Equation (1)}$$

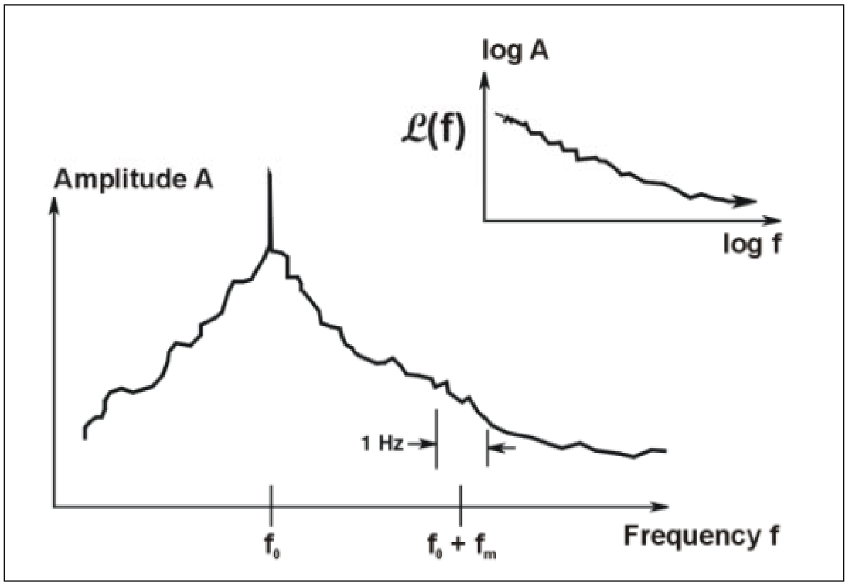

Figure 1. Phase Noise Unit of Measure.

When measuring phase noise directly with an RF spectrum analyzer, the $\mathscr{L}(f)$ ratio is the noise power in a 1 Hz bandwidth at a desired offset frequency f away from the carrier relative to the carrier signal power (Equation 2 and Figure 2) [5].

$$\mathscr{L}(f) = 10\log\left(\frac{\text{Noise Power in 1 Hz BW}}{\text{Total Signal Power}}\right) = P_n[\text{dBm/Hz}] - P_s[\text{dBm}] \qquad \textit{Equation (2)}$$

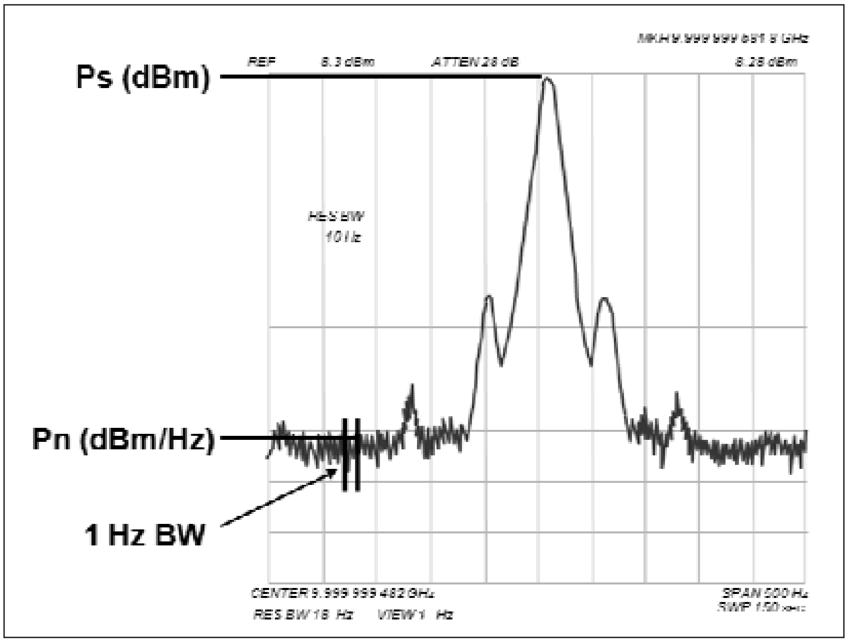

Figure 2. $\mathcal{L}(f)$ *Ratio.*

Another measure of phase noise often used is $S_\phi(f)$, the spectral density of phase fluctuations, expressed in units of phase variance (radians) per 1 Hz bandwidth as given by Equation 3 [6].

$$S_\phi(f) = \frac{\Delta\phi^2_{\text{rms}}(f)}{\text{BW used to measure } \Delta\phi_{\text{rms}}} \quad \left[\frac{\text{rad}^2}{\text{Hz}}\right] \qquad \textit{Equation (3)}$$

$S_\phi(f)$ can also be expressed in terms of dB when referenced to $1 \text{ rad}/\sqrt{\text{Hz}}$.

When the total phase fluctuation in the modulation sideband is much less than 1 radian, $\mathcal{L}(f)$ can be directly related to $S_\phi(f)$:

$$\mathcal{L}(f) = \frac{1}{2} S_\phi(f) \qquad \textit{Equation (4)}$$

However, when the phase variation exceeds the small angle criterion, the historical $\mathcal{L}(f)$ becomes confusing because it's possible to have phase noise density values which are larger than 0 dBc/Hz even though the power in the modulation sideband is not greater than the carrier power.

To eliminate the confusion, the historical definition of $\mathcal{L}(f)$ has been changed to Equation 4. The new definition allows $\mathcal{L}(f)$ being greater than 0 dB. Since $S_\phi(f)$ in dB is relative to 1 radian, results greater 0 dB simply mean the phase variations being measured are larger than 1 radian.

Measurements of $\mathcal{L}(f)$ with a spectrum analyzer typically measure phase noise when the phase variation is much less than 1 radian. On the other hand, phase noise measurement systems with phase detectors measure $S_\phi(f)$, which allows the phase variation to exceed the small angle criterion. The traceabilities of the two measurement methods are introduced below.

3.2. Traceability of Phase Noise Measured with a Spectrum Analyzer

The most direct and probably the oldest method for phase noise measurement of oscillators is the direct spectrum technique. Here the signal from the unit-under-test (UUT) is input to a spectrum analyzer tuned to the UUT frequency. The sideband noise power can be directly measured and compared to the carrier signal power to obtain $\mathcal{L}(f)$[7]. The measurement equation is as Equation 2.

Theoretically this approach measures both the total sideband noise power of AM noise and phase noise. However, in many cases, due to the manner in which the signal is produced, phase noise sidebands are dominant. If the AM noise is much less than the phase noise (typically 10 dB will suffice), the measurement result can be considered a good approximation of phase noise.

From the block diagram in Figure 3 it is seen that the major contributors to MU come from the bandwidth normalization of the IF filter (i.e., the deviation of the IF filter from an ideal rectangular filter) and the amplitude measurement accuracy of the spectrum analyzer. The latter specifically includes the analyzer's scale fidelity, reference level accuracy, frequency response, resolution bandwidth (RBW) switching uncertainty, etc.

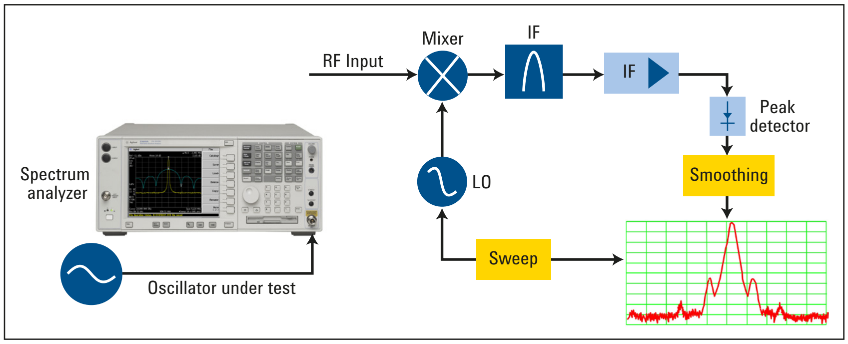

Figure 3. Phase Noise Measurement with Spectrum Analyzers: Setup & Block Diagram.

A spectrum analyzer is normally calibrated periodically against its specifications. The standards for calibration of the aforementioned characteristics include frequency counters, microwave signal generators, RF power sensors and power meters, step attenuators and power splitters. Therefore, phase noise measured by spectrum analyzers can be traced to frequency, RF power (calibration factor), attenuation (transmission coefficient), and reflection coefficient.

3.3. Traceability of Phase Noise Measured with a Phase Detector & a Reference Source

To separate phase noise from AM noise, a phase detector is required. The phase detector receives two input signals and converts their phase difference into a voltage at its output [7, 8]. Any phase fluctuation will result in an output voltage fluctuation as shown in Equation 5.

$$\Delta V_{RMS}(f) = K_{\phi}\Delta\phi_{RMS}(f) \qquad \textit{Equation (5)}$$

One of the most widely used techniques based on the phase detector concept utilizes a reference source and a phase-locked-loop (PLL) as shown in Figure 4 below [8].

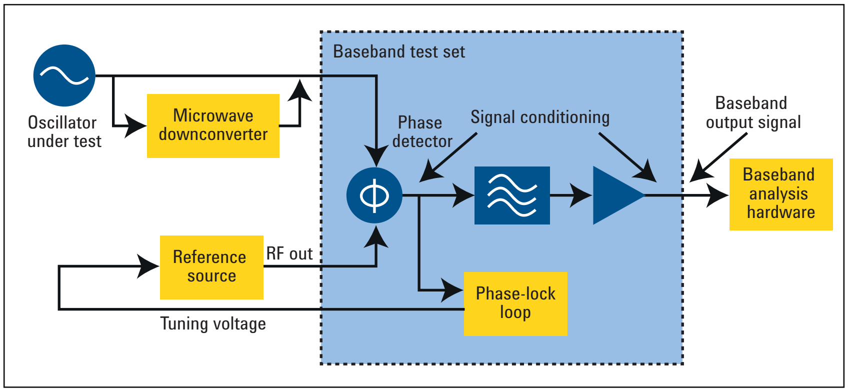

Figure 4. Phase Noise Measurement System with Phase Detector & Reference Source/PLL: Block Diagram.

The PLL is used to control the reference source and establish phase quadrature at the phase detector. The phase noise that is measured at the phase detector is the sum of the mean square phase fluctuations. The reference source has much less phase noise than the UUT so the measurement results reflect the higher phase noise of the UUT. The baseband signal is normally filtered, amplified and input to a baseband spectrum analyzer. A low-noise microwave downconverter can be used to translate the carrier frequency of the UUT to a lower IF frequency for measurement purposes.

The major contributors to MU of the system include the phase noise floor of the downconverter and the reference source (when the phase noise-under-test is close to the floors), the AC/RF measurement accuracy and the internal noise source floor of the baseband analyzer, the PLL suppression calibration factors (when the offset frequency is as small as falling into the PLL bandwidth), the phase difference measurement accuracy and the noise floor of the baseband test set, as well as cable mismatch and signal path flatness (when offset frequency increases).

The calibration of the characteristics mentioned above of the reference source, the downconverter (when used), the baseband test set and the baseband analyzer involve frequency counters, AC and RF signal generators, attenuators, splitters, power sensors and meters, and noise sources and meters. Therefore, with this method phase noise is traced to frequency, AC voltage, RF power (calibration factor), noise, attenuation (transmission coefficient), and reflection coefficient.

4. Summary

Many calibration laboratories, having had some traceable quantities, only provide calibration for the same quantities where direct traceability is shown. However, if a quantity can be derived from the traceable ones, the traceability of the new quantity may also be established.

The term "*derived quantity*" on a traceability chain is different than that defined in the SI Brochure since it is not necessarily derived from or traced to the SI "*base quantities*" directly; being derived from and linked to traceable quantities that are readily available is enough. Even an SI base quantity (e.g., electric current) can be derived from (and thus traced to) other quantities (e.g., voltage and resistance).

Generally speaking, the approach of establishing traceability for a derived quantity is similar to that for a directly traced quantity. Details of measurement method, setup, connections, measurement equations and MU analysis should be documented. However, attention should be specially paid to the translation from the physical principle of the measurement technique to the final MU analysis. Full understanding to the measurement is essential to ensure that all significant MU contributors are appropriately considered.

As an example, phase noise can be traced to different quantities corresponding to different measurement techniques. With the direct spectrum method it can be traced to frequency, RF power, attenuation (transmission coefficient) and reflection coefficient. With the phase detector and reference source technique the traced quantities include frequency, AC voltage, RF power (calibration factor), noise, attenuation (transmission coefficient) and reflection coefficient.

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